

Negative Dimensionality and General Schemas Theory

The Advance of the Systems Engineering Discipline through an extension of Systems Theory

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Introduction

In the previous papers in this series we have described General Schemas Theory and given a representation of it using Pascal's triangle. We have embedded it in a broader framework that includes Logic and Mathesis and attempted to show how that framework needs extension in order to serve to elucidate General Schemas Theory properly. Then we attempted to adumbrate this framework showing how it relates to Systems Theory and Systems Engineering as sister disciplines. In this paper we will dive more deeply and consider the key concept of negative dimensionality. A search of several databases has revealed that no one seems to have considered this possibility previously, which is

odd because they seemed to have considered everything else under the sun as possibilities. So there must be someone within the tradition who has addressed the idea previously, but I have not been able to find that reference as yet. So I intend here to explore this territory myself as it is a key idea that comes out of the tie of general schemas theory to the representation of Pascal's triangle. That tie is rooted in Plato's Timaeus and gives general schemas theory a precision that it would lack otherwise as we might expect from a theory tied to mathematical underpinnings. Our major contention is that schemas have been ignored in our tradition despite their early mention by Plato and their elaboration by Kant. We have a well developed Logic in myriad forms and a well developed Mathesis in myriad forms but when it comes to schemas we have only a series of vague ideas which we can begin to make precise by the tie to Pascal's triangle. But that leads to an interesting conclusion that has not been explored at all in our tradition, which is whether Pascal's triangle has a negative image. We know there is negative matter, called anti-matter. There is negative energy, which is constrained by very severe limits. There is negative entropy, which are the basis of dissipative structures in far from equilibrium environments. And recently in one of my papers I have talked about negative information and its relation to the reflexive autopoietic system. But when you look up each of these concepts on the internet you find something that someone has to say about it one way or another. But negative dimension is a concept that is just not developed¹. I guess the reason why is that it is ultra counter intuitive. But then a lot of other things are

¹ Well not developed as a bonafide concept. See The Adventures of Discriminant Boy In The Negative Dimension. <http://imaginary.puzzling.org/>

counter intuitive and have turned out to be true, like quantum mechanics for instance. So lets begin slowly and from the beginning and see where this new adventure takes us. The idea here is to focus on the most problematic idea underlying General Schemas Theory in order to attempt to ground that theory more rigorously.

The reasoning that takes us to negative dimensionality is quite simple. Once it is realized that there is a connection between schemas and dimensions and that Pascal's triangle is the key to understanding the unfolding of dimensions, as it presents us with the minimal solid in each dimension, then it is merely a matter of wondering what would happen if we step up or down the dimensional ladder. If we step up that ladder we go beyond the tenth dimension which takes us beyond the pluriverse into unknown territory. But if we step down the triange we eventually hit zero which must be beyond the one that is the first term in the triangle. Well if there are integers with positive and negative values on either side of zero, then we might expect a negative Pascal Triangle beyond zero to balance the positive Pascal triangle this side of zero. In other words there is probably a balance of negative and positive triangles just like there is a balance of negative and positive numbers. Once we accept this premise then it becomes clear that what negative dimensionality is made of is hyper-complex algebras. In other words, models of interpenetration of everything that has positive dimension. This suddenly gives us a place for the sources to exist, in the meta-system of dimensionality. Dimensionality has an origin of zero dimension. It has a boundary which is each dimensional transition or horizon. It has an arena which is each successive dimension itself. All that is missing is the source aspect of

the meta-system. But when negative dimension is posited then that becomes the place where the source templates arise from and we have a complete meta-system model in dimensionality. This suddenly connects with Platonic meta-physics that talks about the "Forms" which are in some transcendental "dimension". What if Plato's forms are in negative dimension? Have we really considered carefully what the difference between the transcendental realm posited by Plato and dimensionality of the world actually means. Could that difference merely mean that the forms are in a negative dimension? I think this is an interesting possibility to explore. So we will explore it here.

Negative Dimension and Pascal's Triangle

Lets start again and progress more carefully. Everyone is agreed on the number line with natural numbers, integers, rational numbers, real numbers, imaginary numbers and even hyper-imaginary numbers. It is the structure of mathematical operations that forces each of these transitions on us one by one. We need each of these kinds of numbers in order to solve problems that arise in calculations. We easily accept the necessity of negativity to ease the problems of adding and subtracting past zero. However, when we talk about the world we generally see positive values, and only recently have we come to recognize things like negative matter, negative energy, negative entropy and negative information. So slowly we realize that going past zero has some meaning for certain quantities in the physical world. However, intuitively it seems that there is little meaning in negative dimension because when we hit the zero dimension we seem to come up against a wall past which we cannot experience anything.

But perhaps the problem is that we have not thought deeply enough about this wall we hit when we attempt to understand what might be past the zeroth dimension in the direction of negative dimensionality.

For my part I came to explore this territory by considering the question of the difference between Buddhist Emptiness and Taoist Void. They are definitely very different concepts but still related. Then I hit on the question of whether there is a difference between even and odd zero. There is a controversy whether zero is odd or even in number theory. A very low key controversy but still there is disagreement on this issue. I realized that the difference between even zero and odd zero might be related to the difference between Buddhist Emptiness and Taoist Void. In fact I believe that even zero is an image of emptiness and odd zero is an image of the void. But then the question becomes, where is odd zero. Because on the number line we see only the odd zero between positive one and negative one. If we look at Pascal's triangle we see **1** then **1...1** then **1 2 1** which is a line. Thus **1...1** must really be **1.0.1** as origin and dimensionless point. **1 3 3 1** is the triangle of course. But if we read in the opposite direction toward negative dimensionality then we run into the origin point **1.0.1** and then the two limits fuse to give a oneness that encompasses the whole unfolding 2^n series which is **1**. It must be that on the other side of this **1** there is another **0** different from the origin. And if there is an odd zero past **1** then there must be a **-1** which acts as source which is the complementarity of the origin in the meta-system. Now this **-1** is very interesting because in the number line **-1** as an integer is a singularity out of which unfold all the hyper-imaginary numbers. So we can consider that **-1** must also be such a

singularity. And sure enough we see that what unfolds from it is the whole series of hyper-imaginary numbers. First there is **-1...i**, then **-1.jk.i**, then **-1.ijk.IJK.E**, etc. One of the roles of Pascal's triangle is to differentiate the levels of unfolding and articulation of the hyper-complex algebras. What we are seeing is that there is a difference in position between even and odd zero. Even zero is at the origin and at the center of the number line defined by **1.2.1**. Odd Zero is between the stalagmite of the positive Pascal Triangle we all know and love, it was actually discovered by Khu Shijie² around 1303, and the stalactite of the negative Pascal triangle. Once we go past odd zero we hit the singularity from which unfolds the hypercomplex algebras. That is the real source of imaginary numbers not the **-1** on the integer line. The **-1** on the integer line is merely a stand in for the greater singularity of **-1** that stands outside the number line beyond odd zero. Once you realize that hyper-complex algebras are merely models of interpenetration, then it becomes clear that what is beyond the void is really just the sea of interpenetration that underlies everything. We know that anything that was together at one time is intimately linked by a spooky action at a distance according to Bells Theorem. Since everything was squashed together originally before the big bang then that means that everything in the universe is intimately linked by spooky action at a distance. That linkage takes place in negative dimensionality though the interpenetration modeled by the hyper-complex algebras. Odd zero is a wall to us and beyond that wall is the ground of interpenetration of all things that unfolds from the source singularity **-1**. That source singularity is what stands just prior to the Big Bang physically, and which stands

² <http://www.roma.unisa.edu.au/07305/pascal.htm>

beyond positive dimensionality as the completion of the meta-system of dimensionality. It is probably the source of *quintessence* or the dark energy that is accelerating the matter of the universe away from each other as spacetime continues to accelerate its expansion. Where ever we look there is just dimensionless space which is even zero. But that unfolds from what is called in China the Great Ultimate where Yin and Yang join which is **1** and that stands just after the void of odd zero. And beyond that there is according to the Tao Te Ching the **-1** of the singularity called gate of the Mysterious Female. This is a cornucopia or source from which everything arises. Things unfold from one to two to three to the myriad things. But prior to the great ultimate one there is the void or Tao and prior to the void there is the Mysterious Female. The Chinese seem to have had a very good picture of the relation of the positive and negative images of Pascal's triangle as it relates as a model to the unfolding of differentiation in existence. That picture becomes more clear when we relate what is said in the Tao Te Ching and our model of the stalactite and stalagmite triangles. Everything in this interpretation revolves around the clarification of the difference between even and odd zero, and the recognition that the **-1** beyond the void of odd zero is the singularity from which unfolds the hyper-complex algebras which are a model of interpenetration. There are three walls here. First there is **1** at which the limits of the interval of spacetime collapse together. Then there is **0** as odd zero and void. Then there is the **-1** mysterious female source of everything behind which all the imaginary realms unfold. These are insuperable barriers for creatures such as we. We won't be traveling to negative dimensionality any time soon. But what we need to recognize is that

we are already engulfed in it because we are within the meta-system of dimensionality already. The sources are just as much a part of that meta-system as the origin, and all the other points we demark within positive dimensions. Socrates and then Plato were the first to point out those sources. But there philosophy was misunderstood as referring to a transcendental realm rather than an immanent one. That is partially because of the change in Aristotle's philosophy from an emphasis on Masses to an emphasis on Sets as the basis of philosophy itself³.

One of the corollaries of this recognition of the positive and negative Pascal triangles as the mathematical order underlying the articulation of the meta-system of dimensionality is that negative dimensional space is not continuous but like the points on the complex plane it is fragmented. Thus there is a fundamental difference between continuous positive dimensions and discontinuous negative dimensions. This phase transition can also be seen in the relation between sets and masses. Positive dimensions lend themselves to mass like descriptions. Positive dimensions are masses of dimensionless points, even the higher dimensions, even points that are simultaneously in different dimensions. On the other hand the negative dimensional points are all unique and lend themselves to set-like descriptions instead of mass-like descriptions. Just like sets the points of negative dimensionality do not add up to anything, there is no emergent properties of negative dimensions just as there is no emergent properties of sets. On the other hand there are emergent properties of masses of dimensionless points as they are encompassed

³ [The Discovery of Things: Aristotle's Categories and Their Context](#) by Wolfgang-Rainer Mann (Princeton UP)

by higher and higher positive dimensions. Each dimension has its own synergy which it expresses by its overdetermination of points, lines, polygons and polytopes. This phase shift from mass to set has profound implications that we seen in Plato's metaphysics which claims that every form is unique, yet it pervades all the particulars associated with it. Pervasion is necessary within continuous masses with boundaries. On the other hand syllogism is necessary with sets of unique particulars. The whole question of Platonic metaphysics is how unique forms come to pervade objects in spacetime. The partition of spacetime, the so called receptacle, is done by the schemas. Once the schemas have partitioned spacetime giving us individual objects then there is the form-copy which allows the properties to be bundled with respect to that envelope of spacetime. Forms which inhabit negative dimensionality are not in any one place. Rather a form in negative dimensions inhabits all of spacetime. Sets are groups of unique properties associated with forms. Sets are the prototype of essences. Any Form within the negative dimensions can touch all of spacetime. The question becomes why certain forms participate in particular schematic envelopes and not others. This is a question that Plato does not answer, rather he merely sets up the two regions and posits that Forms coordinate across the barrier between the two regions so that the right form always is associated with the right schema. However the Buddhists have an answer to this question which has to do with difference. They say that the differences between the parts of the house are their interpenetration. In other words, differences between the different parts of things are the way they fit together and that fitting together is possible because the different things interpenetrate. This is an interesting answer because it means where

ever we see difference in the world, that difference is complementary, and the complementarity is a sign of the interpenetration of all things. We know that the meta-system is composed of complementarities of complementarities of complementarities infinitely deep and arranged according to the structure of the hyper-complex algebras. This complementarity of the meta-system that includes the negative dimensionality shows up in the differences between things that also exhibit complementarity. The differences are constrained by essences and schemas. Essences are bundles of properties and the schemas allow those bundles to be hierarchically or heterarchically organized and fitted together. Differences of kind cannot really fit together in complementary ways unless they are embodied in schemas. So the schemas in positive dimensionality and the essences of kinds in negative dimensionality are reciprocals. Between these two, and between the extremes of mass and set occur the ipsity, i.e. the unique individual as bearer of a kind in a schema which combines the mass and set like properties into a single conglomerate. When we look at Plato's metaphysics through the lens of positive and negative dimensionality it starts to seem more plausible. When we go further and recognize that all of the versions of Plato's metaphysics are images of Special Systems Theory then we begin to have some appreciation of its profundity. Special Systems are merely the poking though into positive dimensions of aspects of negative dimensionality associated with complexions, quaternions and octonions. Other hyper algebras merely give us the complementarities that go infinitely deep with the negative Pascal Triangle. But these first few hyper-complex algebras have special properties which are ultra-efficacious. Plato

was aware of the qualities of the special systems and he has woven hints to their existence throughout his dialogues. One place they appear proximately are in the odd characteristics of his cities. But once his metaphysics as a whole is understood as we can through the broad view given in Allan Silverman's *Dialectic of Essence*, then we see that everything Plato says about the world is conditioned by his knowledge of the Special Systems, a knowledge that was lost in the West but preserved in Taoism, Buddhism and other Eastern philosophical worldviews. Traditional non-western sciences in many instances is based on this knowledge, for instance Chinese Accupuncture or Islamic Homeopathy. Alchemy was originally in the form practiced by Bolos was the study of the efficacies of Special Systems. There are fragments of this knowledge left all over the globe. But for the most part it has been lost in terms of being a global way of approaching existence. The rediscovery of the crucial role played by the Triangle of Pascal is an important step forward in the restitution of this ancient knowledge. Who would have thought that the negation of Pascal's triangle would be related to the understanding of Plato's doctrine of Forms. It also holds a key to understanding Schemas theory because we can now see how it is connected to the theory of kinds. There is a conjunction between positive and negative triangles on either side of Odd Zero or Void. Forms are set like and inhabit negative dimensionality. Schemas cut up spacetime into envelopes. There is a mysterious conjunction between the envelopes and the forms, like the conjunction Plato often talks about between souls and bodies. That mysterious conjunction has to do with the relation between interpenetration and difference. Difference in positive spacetime is the sign of interpenetration between things

through the negative dimensions. Forms in negative dimensionality permeate everywhere. But they only show up their bundled properties within the appropriate kinds of things. But that constraint on where they show up their unique properties of a given form is diacritically determined by all the other forms and their differences, and all the other schemas and their differences embodying forms. Essences are constraints on properties bundled together as a form. But in a back handed way all the other forms constrain any one form to show up where it does in the grand scheme of things. It shows its difference just where that difference is needed and nowhere else. How this conjunction occurs is of course a mystery even to Plato.

There is a problem which is as old as philosophy itself how it is that properties can be distributed through objects in spacetime and still be the same. This problem may be addressed with the aid of the concept of negative dimensionality. This is because there is a reversal between Mass and Set approaches across the divide between positive and negative dimensionality. As we have said the negative dimension is set-like and positive dimension is mass-like in itself, but when we talk about things in spacetime with properties from forms in negative dimension then these two approaches reverse, in a chiasmic way similar to the reversal of logos in physus and physus in logos that allowed us to distinguish logic and schemas. The reversal is this. Objects in spacetime have essences which are setlike, i.e. a set of unique properties bundled together to form a natural complex. But essences are more than merely collections of properties, they act as constraints on properties in their bundle too. So within the mass-like dominion of spacetime of positive dimension there is the set-like

essence which Plato calls the *form-copy*. Each kind of thing within its schematic envelope has a form-copy or essence that participates in a form. Now this participation in a form is mass-like. In other words Forms from negative dimensionality pervade the form-copies associated with schemas, i.e. spacetime envelopes. So there is pervasion of the set-like nature of negative dimension across the void. On the other hand the set like nature of “Forms” come to syllogize the schematized essences of things in spacetime. We use the term syllogize as the opposite of pervasion. Sets have a syllogistic logic associated with them whereas masses have a pervasion logic associated with them. It is easy to see how properties pervade spacetime to show up in specific essences. It is not so easy to see how the set-like quality of negative dimensionality comes to provide the form-copies which serve as essences for schematized things in spacetime. But what we are implying is that the cross over provides a logic in both cases. In the case of the syllogistic logic there is an appeal to a universal, an illusory continuity that is an ideal as the means of limiting syllogizing. In the case of the pervasion logic there is an appeal to a boundary, a distinction or limit of the mass or solution of masses as a means of limiting pervasion. Notice how this complementarity and chiasm of applying the set-like and mass-like approaches differentially to the plenum of positive and negative dimension on the one hand and to the things in positive and negative dimension on the other suddenly clarifies the problem of how properties can pervade spacetime yet still be the same. This is possible because of the introduction of the key terms of universal and boundary. Essences relate to universal ideas as a mediation between them and their source forms. The schemas themselves provide the

envelopes or boundaries which contain these pervasions. This chiasmic complementarity of the differential application of set and mass like approaches to the plenum of negative and positive dimensionality and the things within positive and negative dimensionality is the key to understanding this age old problem in a new light. Plato’s “Forms” to not exist in a transcendental realm but in an immanent realm, the realm of negative dimension. It is immanent because it is all around us in the world we experience where set-like platonic forms participate in each other and properties participate in them. When we look at the set-like forms we are looking at negative dimensionality in our experience. These set-like kinds produce myriad differences in the world but also sameness between the things that mutually participate in the same Form. These differences and samenesses are indicators of the underlying interpenetration of all things. A weaker form of interpenetration is the fittingness of all things into each other. When we see how the natural world seems designed everything fitting together so perfectly we are seeing a face of the deeper interpenetration that is the primary characteristic of negative dimensionality. Negative dimensionality is the ground of interpenetration beneath the differences we experience in the world. The interpenetration is modeled by hypercomplex algebras which embody the relations between mirrors in different complex configurations. Interpenetration is mutual mirroring of everything with everything else. In the negative dimensional mirrors we see the differences between things ramify out and at the same time we see the participation of properties in forms which allows the complementary sameness to extend throughout spacetime in schematic envelopes of the same type. Kindness comes from the source forms in

negative dimensionality. Schematicism is its opposite and that comes from the partitioning of spacetime. Schemas and kinds together describe everything we experience except for the ipsities, the uniquenesses of the individuals which are the non-dual between the nihilistic opposites of too much difference and too much sameness that we see in the set and mass approaches.

Of course, this insight into the nature of the distribution of properties will have to be worked out in more detail and with greater attention to Plato's presentation of his various theories of forms. Just like with schemas and dimensionality, the hyper-complex algebras as models of interpenetration do not capture all the emergent properties of source Forms. In the literature of philosophy these transcendentals of Plato are called "Forms". But we see them as immanent sources. It is confusing because Plato concentrates on one schema the Form and applies it to everything thus making it the primary schema in our tradition. Thus what are sources, i.e. templates of things beyond spacetime in negative dimensionality, become conflated with the schemas to which they are associated. This confusion and conflation must be understood clearly if we are to unravel the complexities of the situation. We take a meta-systemic view of dimensionality. That calls on us to introduce negative dimensionality so there is a place for the sources. These sources are the reserves from which things draw in order to give rise to their properties. As sources in negative dimensionality they can appear anywhere in spacetime. They have access to all of spacetime. The real question becomes how they become limited to certain schemas that have particular kinds of essences. It is by this limitation that we learn that essences are not just arbitrary sets but

constraints on properties. Natural complexes display ranges in constraints on complexes within a specific kind associated with a particular essence. Notice that the form-copy is a repetition of the form. Notice that the embodiment of the form in a schema is a representation of a higher dimensional negative reality in a lower dimensional positive reality. Here again we see that representation and repetition come together in the production of the kinds of things in spacetime what display properties from the same source. It is a complex situation this problematic of how properties become distributed in things of the same kind. To understand it we need to apply many tools and it turns out that these are the same tools needed to understand how the framework of general schemas theory can to be understood.

Pascal Simplicies

The next step is to understand where Pascal's triangle comes from and how it fits with other parts of mathematics. When we look into the literature we see that it stands as something isolated, something of an oddity, between serious mathematics and magic squares. It has many curious features that are in combination inexplicable. Perhaps we can right this impression if we consider the Pascal triangle as the next higher emergent level from the integer number line. This is to say the integer number line is produced by addition and subtraction from zero. Each number is created by adding one or negative one to the last integer. But what mathematicians in general seem to have failed to ask is what is the next higher emergent level from the integer number line. If we ask that question then we can see that where the line is two dimensional Pascal's triangle is a two dimensional array of numbers. But that is not a standard array because it expands as it unfolds. Notice that the integer

sequence has no room to expand because it is one dimensional. But once we move up to a two dimensional array then we see that Pascal's triangle expands in an array with each line. Each line is produced just like on the integer number line by addition, however in this case we have diagonal addition of two previous values from the previous line of unfolding. If we extend this analogy we realize that there exist Pascal Tetrahedrons which are three dimensional expansions, and Pascal Pentahedrons that are four dimensional, etc. In other words there are Pascal Simplicies⁴ for each dimension which expand into arrays in those dimensions. Now here is the curious thing about the Simplicies of the various dimensions which are each generalizations of the line at the various emergent levels, they track the geometrical minimal polytopes for each dimension that Pascal's triangle itself defines. In other words Pascal's Triangle is self defining in the sense that it produces images of the simplest Platonic Solid in each emergent dimension, and the successive simplicies which are the expansions of Pascal's triangle itself follow those same forms for each dimension. So this gives us an interesting picture of the place of Pascal's triangle in mathematics. The triangle is the next step up from the integer line expressed in two dimensions and with its own emergent properties. Those properties cause the triangle to expand into the two dimensional array just like a fractal cellular automata. In fact the odd numbers form a fractal organization named after Sierpinski. The two dimensional line is a Pascal Triangle. The three dimensional integer line is the Pascal Tetrahedron, the four dimensional integer array is the Pascal Pentahedron, etc. The Pascal Triangle defines the simplicies for higher order integer

expanding arrays analogous to the integer line. In other words the expansion of the integer line to higher levels is self-defining in that the Pascal Triangle which is a generalization of the integer line into two dimensions defines all the higher simplicities that are the next emergent steps of the Pascal triangle itself. This is an image of self-production, where the Pascal triangle is laying down the template of its own emergent unfolding.

We would be remiss if it were not mentioned the connection between the Tetrakys and Pascal's triangle. The Tetrakys is a Pythagorean triangle of ten dots. This triangle can be seen as the tip of the pascal triangle, the minimal system of elements from which the entire triangle is generated. It also corresponds to the position of the special systems in the negative dimensional triangle. Elsewhere we have postulated that Being's fragments, i.e. Pure, Process, Hyper and Wild Being can be combined in a multilith in various ways. It turns out that there are twenty four such permutations. Twenty four is $1*2*3*4$ which is the multiplicative Tetrakys rather than the additive one we see in the triangle of Pascal. Another interesting connection is to the surreal numbers. These numbers are generated by using up and down arrows and tracking a path through a progressive bisection of up and down arrows that represent moves in a game. We can imagine that the Tetrakys is related both to the surreal numbers and later to the triangle of Pascal being the minimal generating unit. The surreal numbers is a progressive bisection so at each level there are 2^n elements. The triangle of Pascal describes at each of its levels a 2^n system. Thus we can see that there is a relation between the Pascal triangle as a generalization of the integer line and the surreal numbers. Each row added to the Pascal Triangle is a

⁴ <http://people.ucsc.edu/~erowland/pascal.html>

new 2^n system which flows from the progressive bisection of the surreal numbers. The interesting thing about the surreal numbers is that it gives with minimal rules many types of numbers at once rather than their having to be build up one by one. The problem with the surreal numbers is that they have holes in them. Surreal numbers are the orthogonal number system to the generalized integer line of the Pascal triangle. Surreal numbers are the natural numbering of the meta-system. Meta-systems have holes in them. They are wholes less than the sum of their parts, i.e. wholes with holes like a sponge. Surreal numbers have this property. But each stage of the unfolding of the Pascal triangle is in lock step with the unfolding of the surreal numbers by progressive bisection. Thus the Pascal triangle mediates between the normal number system which is fragmented into classes produced by numerical operations that represents the 'system' and the meta-system of the surreal numbers. This connection between the number system produced by arithmetical operations which is fragmented into classes but ultimately continuous, and the number meta-system produced by imitating the moves of a game which is ultimately not continuous through Pascal's triangle is very interesting. Pascal's triangle is a kind of intermediary poised at the level of unfolding of the integer prior to the arising of the rational and real numbers but after the arising of zero and the negative numbers from the natural numbers. We might say that at this intermediate point in the unfolding of the number system there is a generalization of the integers into the Pascal triangle which then connects to the surreal numbers which is an alternative way of generating numbers which is the dual of the number system which might be thought of as the number meta-system. This intermediate between the system and the meta-system like

the special systems has the quality of being self-producing in as much as it creates the template for its own expansion by defining the simplicies that it would expand into emergently. So Pascal's triangle is a special systems image. Its expansion into the array of places by diagonal addition is dissipative. There are two such dissipative waves from zero and they move into positive and negative dimensions balancing each other but with very different characteristics. One defines minimal solids of each dimension thus defining those dimensions while the other defines the expansion of the hyper-complex algebras. As we saw with our analysis of the role of the blending of kinds and schemas these two wave fronts of the expanding Pascal triangles intermingle to produce a complete meta-system formation with source, origin, arena and boundary all defined within symbiosis of negative and positive Pascal triangles interweaving. However, this brings us to another interesting point, which is the fact that there can be a reflexive level to the Pascal Triangles if we consider that from the odd zero of the void their arises four different Pascal triangles that fill the plane, two negative/positive pairs at right angles to each other. This means that out of the void four 1s arise. This is a minimal system according to B. Fuller. Two conjuncted autopoietic systems make a reflexive system. Four conjuncted dissipative systems make a reflexive system. Four Pascal Triangles all produced from the odd zero of emptiness all dissipating toward infinity at 90 degree rotations from each other on the Cartesian coordinates could be seen as an image of the reflexive special system. This might be an image of the causality of a Yang celestial cause hitting a Yin surface with four different receptivities. There would be four causal chains in four directions actualized. Like dropping four stones in a pool of water

they would set up an interference pattern of waves in the water. Thus we could see that there is really two positive and negative Pascal Triangle arrays orthogonal to each other which might interact. We need to consider what that interaction might entail at the reflexive level. Just as two lines are at right angles to each other, so two positive/negative Pascal triangle configurations may be at right angles to each other. Yet we know that positive and negative branches of the Pascal Triangle configuration interact, so we might suspect that the two right angled configurations might interact as well to give a higher order emergent phenomena. Could that interaction give rise to the next higher order simplicies? Pascal's triangle itself has already defined the plane as two crossing lines of ones and negative ones. When we add another orthogonal triangle we have defined the plane twice which gives the possibility of the second plane rotating out to define the next higher dimension. Once we have defined the next higher dimension then a new simplicies can be formed in that dimension, and so on up the scale of minimal solids. So the reflexive emergent quality that appears with the combination of two Pascal hour glasses is the possibility of moving up to the next dimension of Pascal simplicies. Thus the Pascal Hour glass has built into it its own generation of the next emergent dimensional level. This is another autopoietic like characteristic. The two dimensional twin hour glasses provide the mechanism for creating its own next emergent unfolding. That unfolding is synergetic because higher dimensional polytopes are synergetic. In that synergy is embedded their emergent properties. New things become possible with each new dimension which were not possible in the lower dimensionality. Thus we posit that Pascal's triangle is a non-dual between the continuous number system of normal math and

the surreal discontinuous number system of exotic math which includes infinitesimals. Pascal's triangle is a generalization of the number line and it has stages related to the special systems. Its cellular automata like expansion is dissipative. The interaction of positive and negative dimensions in autopoietic and two Pascal hour glasses combine to give rise to a reflection of the reflexive special system that generates the next higher Pascal simplicies. Poised between the system of real numbers and the meta-system of surreal numbers is the special system of the positive and negative Pascal triangles. We can imagine that when one hourglass rotates to produce a second hourglass that the second one might occupy an imaginary plane in relation to the first with i and $-i$ as original starting points. In this way we would imagine the complex plane as arising after the complex numbers. It is interesting that -1 is beyond odd zero void as the singularity out of which the hyper complex numbers unfold in their elements. But the complex plane is the result of the 90 degree rotation of the Pascal Hour Glass to produce the complex plane. That complex plane can collapse into a third orthogonal coordinate to define three dimensional space. In that space the three dimensional Pascal tetrahedron could form. This would mean that the complex plane was an intermediate between the two dimensionality and the unfolding of three dimensionality. The same would be true for each higher configuration of simplexes.

My hypothesis is that just as the difference between positive and negative dimensionality appears as a reciprocal chiasm between mass and set like properties of their respective regions that with respect to the rotated hour glass associated with positive and negative i that this is associated with a reciprocal chiasm

between Reserve and Field. Reserve and Field are the complementary opposite pair to the Mass and Set pair. They are taken from Physics and each one has an associated logic and an associated mathematical category which has gone unrecognized up until this point. But these categories have had to be developed for Physics to make progress. Just as Thomas Etter in his Link Theory things that the core part of quantum mechanics is really part of probability theory, here we are advancing the idea that some of the core parts of normal physics are also part of mathematics rather than physics. Fields are everywhere in physics. It seems strange that there is no field mathematical category. The term field as it is now used in mathematics does not mean the same thing. What we are talking about is not something related to a mathematical ring, which is one step up from group theory. Rather we are talking about two more basic mathematical categories. We think there should be four fundamental mathematical categories including Set, Mass, Field and Reserve. A Field is an object like electromagnetism with invisible potential lines that vary in intensity at each point in spacetime and its logic has to do with transformations. A Reserve is an object which is conserved, like energy, and its logic has to do with accounting. If we generalize the Field and the Reserve we get a complementary pair very much like Set and Mass which is complementary to that pair. Our hypothesis is that when you rotate the hour glass of positive and negative dimension 90 degrees then you get a trade off between Field and Reserve as the emergent properties of the i and $-i$ hour glass. We would like to note that just as the Set mathematical category is related to Pure Being, Mass category is related to Process Being, Reserve category is related to Hyper Being, and Field category is related to Wild

Being. Each of the basic mathematical categories are related to different fragments of Being. Now as we said this means that the plenum of i is like the field and the plenum of $-i$ is like the reserve. In fact i is used in the description of many fields. For instance electricity is described by using i because it is like a four dimensional vortex. There are many uses of complex numbers in mathematics and we posit that when ever they are used they are either describing a field or a reserve. The difference is that the field has variations of intensities of forces at each point in spacetime. The reserve on the other hand may go into a completely potential state that must be accounted for as with energy. So reserves can be completely non-manifest where as fields if they occur are invisible but manifest. Both fields and reserves can be described mathematically without any connection to physical properties. There are many types of fields in physics and many types of conserved quantities that form a reserve. If i is associated with the field and $-i$ associated with the reserve, then we would expect the field plenum to be substantially different from the reserve plenum as one moves into imaginary positive dimensions and the other into imaginary negative dimensions forming as second hour glass. We thought we were going wild just suggesting that there were negative dimensions. Now we are suggesting that there are imaginary dimensions of the positive and negative varieties. In this way the complementarity between the various kinds of Being is explained. Process and Pure Being are complementary and Wild and Hyper Being are complementary. And these form two complementary complementarities. The latter two are very strange in deed but have strange similarities to the concepts of field and reserve in physics. This is what suggests the idea that these are basic mathematical

categories rather than merely physical ways of thinking about entities. The fact that there are many types of reserve and many types of fields suggests that the concepts should be generalized into mathematical categories. Fields are continuous but with invisible lines of force. Reserves are discontinuous because the conserved quantity goes through discrete transformations. Reserves are about accounting for the conserved quantity across discontinuous transformations, but Fields are about continuous transformations of the lines of force. Both fields and reserves are invisible. Sets and Masses organize visible phenomena. Reserves and Fields organize invisible phenomena. So the difference of the 90 degree turn is between visible and invisible. Fields are associated with intensities and propensities. Reserves are associated with potentials and possibilities. One is related to the forces between things. The other is related to the tracking of conserved quantities through various transformations. The reserve is the total conserved quantity. The field is the totality of intensities in spacetime. The field organizes the things in spacetime. The reserve organizes things in timespace. The reserve is more like a source, the field is more like a boundary, just like the Mass is like an arena and the set is like an origin. In other words these basic mathematical categories are organized as a meta-system governing the things within spacetime. So if we think of imaginary positive dimensions as fields and imaginary negative dimensions as reserves then we can begin to think about the chiasm which applies to the things in these plenums. In other words a thing in a field when it is transformed may be tied to a reserve and thus have a conserved quantity that can be accounted for. On the other hand a thing in a reserve when it is transformed may appear in a field of force and be tied to a field and thus

have continuous pressure of that force on it as it moves through spacetime. The forces are tied to the spacetime while the reserves are tied to the things in spacetime. We see forces by the movements of the things in spacetime. We see reserves by the transformations in the things in spacetime. These are much more physical ideas than those of Set and Mass. In physics we attempt to model Fields that entrap things and we attempt to account for the conserved quantities as things are transformed. Fields are related to *encompassing* and Reserves to *bearing*, just as Masses are related to *grasping* and Sets to *pointing*. The fact that we discover fields and reserves in physics may not be because they are objectively there in the phenomena, but because these are basic aspects of mathesis. In mathematics infinitesimals have been banned to non-standard analysis but they are just what we need to understand fields and they are part of the surreal numbers. In mathematics holes as quanta have been banned to surreal numbers but they are just what we need to understand reserves. Both field and reserve appear as aspects of surreal numbers. Set and Mass are more like the classical number system while Reserve and Field is more like the surreal number system. Pascal's two hour glasses connect the classical number system with the surreal number system so we might expect one part of it to be more like one and the other stranger part to be more like the other. We then are forced to consider reserve objects in field plenums and field objects within reserve plenums on the model of the chiasm between positive and negative dimensionality given previously in this paper. A reserve object in a field plenum is something non-manifest in an invisible field of forces. A field object in a reserve plenum is something invisible in a unmanifest state. The key to understanding

this is to understand that transformations may be continuous or discontinuous. Field transformations are continuous and reserve transformations are discontinuous. In a conserved quantity there are stages of transformation and the conserved quantity stays the same throughout the transformations. On the other hand fields vary continuously over spacetime whether objects are present or not. A field is continuous if it is there. But if it is not there then there is a discontinuous boundary of the field as a whole. On the other hand with the conserved quantity the conservation is the continuity across many discontinuities. Slowly we begin to see the reciprocity between the field and the reserve. Where one is continuous the other is discontinuous and vice versa with regard to their transformations. With set and mass there was an emphasis on the persistence of the instance or particular. With reserve and field there is an emphasis on the continuity and discontinuity of transformations.

All four of the basic mathematical categories, i.e. Set, Mass, Field, and Reserve come together and intersect at each point in spacetime. If we follow Thomas Etter and his link theory then we need to recognize that each possibility defined by a 2^n structure can have a different probability count that is either positive, zero, negative, or imaginary. We can think of a given 2^n structure of a certain order n as something physical, something mathematical, or something logical. That is to say we can think of bits, or distinctions, we can think of binary bases, or we can think of Boolean or some other kind of more exotic logic. All these things are positive articulations of 2^n systems of a certain order. What is opposite these is the hyper-complex algebras which is a model of interpenetration. The positive 2^n systems can have n^2 relations, i.e.

external relations between elements built up out of distinctions, or bits, or bifurcations, or logical terms. The hyper complex algebras appear as degrees of impossibility, or necessity. Interpenetration is a limit. We can think about it like the degrees of non-computability. But following Etter we need to allow the possibility of imaginary probabilities both positive and negative as well. These are related to invisibles. In the one case the invisibility is manifest and in the other case it is non-manifest. Thus I conceive of Etter's Link Theory as being an intersection of the four hour glass 2^n systems where the actuality is drawn either from the Set, Mass, Field or Reserve domains. If different possibilities for a particular 2^n system are drawn from different lobes of the hour glass then we get a mixed exotic probability that can be negative, imaginary or imaginary and negative both as well as positive. It is Etter's Link Theory that allows us to consider Quantum Mechanics as being a macro-phenomena rather than just a micro phenomena. It is our projection of Being on things that renders them classical and Newtonian. That is a limitation to just seeing positive probabilities. But given any generic 2^n system, we can draw it from any of the four lobes of the dual hour glasses. It is this extension of Pascal's triangle into the negative and imaginary realms that makes possible for us to understand the generality of Etter's Link Theory. Given a generic 2^n system then we can think of different combinations as reaching out to 2^n specifications in each of the hour glass lobes in order to be instantiated. It is not just that there is a set of possibilities that somehow are thrust into negative and imaginary probabilities, but rather that the whole link system can partake of Set, Mass, Field or Reserve lobes of the two Pascal Hour Glasses. Each actualized possibility can appear as a particular in a Set, an instance in a

Mass, a potential in a Reserve, or an intensity in a Field. If it is partaking of the Field or Reserve then it is invisible. If it is an instance in a mass it has lost its particularity into the plenum of the mass. We may think that actualities are always identifiable particulars. But in fact actualities may submerge in a mass, may become invisible as intensities of forces, may become unmanifest as reserve potentials. That a particular $2n$ Link System might partake in all four of these regimes is something that our philosophy of science does not encompass at this time, but which it may encompass in the future as we realize that the quantum mechanical nature of things goes from the micro through the meso to the macro levels of existence. Everything is quantum mechanical and anything that does not appear so is merely an illusion created by the projection of Being within the Western worldview. It is only the Western worldview that posits the Copenhagen boundary. Many sophisticated traditional cosmologies have always understood the nature of existence as quantum mechanical from top to bottom. That is why a book like the Tao of Physics by Capra could be written.

Theory of the Generalized Integer Sequence and General Schemas Theory

In this essay we have proposed a radical new theory of the mathematical representation of the schema. That theory moves beyond Thomas Ethers concept of Link Theory. It proposes that we take seriously not just negative dimensions but also imaginary positive and negative dimensions. It proposes that we extend Pascal's Triangle into an hour glass that represents both the positive stalagmite and the negative stalactite as well as the concept of rotating these 90 degrees into the imaginary plane. Thus we have two Pascal hour glasses at right angles to each other. We

then use the basic mathematical categories of Set, Mass, Reserve and Field that correspond to the fragments of Being, i.e. Pure, Process, Hyper and Wild to interpret the interaction between these clover leaves⁵ of the two hour glasses. Each pair of Basic Mathematical Categories have complementary chiasmic embeddings into each other and thus explain the transition from the description of plena to objects within the plena. Two of these basic mathematical categories are taken from physics, generalized from physical fields and physical conserved quantities. Set theory is already the most basic mathematical category. To that we add its complementary opposite Mass. These four basic mathematical categories and their associated logics give meaning to the lobes of the two orthogonal Pascal hour glasses. They give meaning to what would merely be abstract negative and imaginary proliferations of numbers in the Pascal series and in Ethers exotic probability model called Link Theory. Our theory of Schema is that they are emergent properties that arise between dimensions defined by the Pascal Triangle layers. However, we note that the Pascal Triangle has some odd properties that lend themselves to attempting to understand the emergent properties of the schema, at least their progression and limits. We will explore this in more detail elsewhere. But an example is how a number is a monad. That monad is used to compose the pattern of the line in the Pascal Triangle. The series of lines give us a two dimensional pattern. That two dimensional pattern defines the tetrahedron as an object. The next higher generalization of the Pascal Triangle is the Pascal Tetrahedron that fits in that form. Notice that this gives us a hint that as the Pascal Triangle unfolds it is self-defining. The

⁵ Sic, also assuming four leaf clovers.

question arises whether each level of this unfolding of the simplicies produces new properties and whether we can see the pattern of two schemas per dimension and two dimensions per schema that we attribute to the hierarchy of schemas. This is an open question. However, it should be noted that even if negative and imaginary dimensions do not exist, still the positive dimensions defined by the Pascal Triangle define the schemas. And even if the Pascal Triangle itself does not exhibit the emergent properties of the hierarchy of schemas it still exists as a representation of a limit for that hierarchy of the schemas. Therefore although the theory expressed here is very wild, even its conservative form is useful if we are forced back on just positive dimensions defined by the normal Pascal Triangle. But I think the extensions outlined here are interesting and useful. And hopefully a further exploration of the Pascal Simplicies will reveal emergent properties to them which will strengthen our understanding of the limits of the emergent ontological hierarchy of the schemas. But should it turn out that there are no other properties of the Pascal dual hour glasses than those we have already proposed then this is enough for our purpose which is to give a mathematical representation to the schemas via the unfolding dimensionalities of the Pascal triangle. Negative dimensionality is our most pretentious claim. I think that this essay has shown its plausibility, and it has led to an indication of imaginary dimensionalities as well that are worth exploring. It has given us a bridge back to Eppers Link Theory which is unexpected. We have shown that Pascal's Triangle is a generalization of the integer line and that it connects normal number theory with standard analysis to surreal number theory with non-standard analysis with infinitesimals. It is unclear whether

mathematicians understood this role played by the Pascal Triangle previously. It gives us a model of the special systems between system and meta-system as we moved from Pascal Triangle, to Pascal Hour Glass, to Pascal four leaf clover and built a model of each of the special systems. That analysis roots the Pascal Triangle and its extensions in the interpenetrating substrate that the special systems model and thus explains some of the extra-ordinary features of the Pascal Triangle. Another horizon for future research is whether these odd features of the Pascal Triangle can be explained based on our interpretation. Certainly the over-determination of the Pascal Triangle giving rise to its many uses is explained by the fact that different uses of 2^n systems are brought together by its definition. It could be that the many odd features of the Pascal Triangle link together and may be explained by an analysis that takes our interpretation of the meaning of the Pascal Triangle in the greater context we have tried to paint. However, these explorations will be left to another time. The key point we are trying to establish here is the importance of negative dimensionality and the extension of the Pascal Triangle into that dimensionality in the form of the Pascal hour glass. The further extension into the Pascal Clover Leaf formation notwithstanding we believe that the recognition of this primary extension is very important for our argument because it links the hyper-complex algebras and their unfolding directly to the unfolding of positive dimensionality to produce a whole meta-systemic model. Having a whole meta-systemic model for dimensionality is very important because it is the cross talk between negative and positive dimensions at each layer of 2^n unfolding that allows us to understand the difference between interpenetration internally and intrapenetration externally. It

shows that the hyper-complex unfolding is integral to the unfolding of the schemas as links between dimensions and as pairs within dimensions. It is this linkage that leads to the schemas having their own reflexive autopoietic dissipative structure. The schemas are dissipative in as much as they are projections from the ontological to the ontic by Dasein. They define the ecstatic overflowing of Dasein as a dimensional overflowing into the world and beyond. Thus the schemas are a dissipative ordering of the world by Dasein. Next we have already noted elsewhere that the schemas form an autopoietic ring. Each schema is created by the conjunction of the two schemas on either side of any given schema. And the schemas seem to cycle around in a circle with the facet connecting to the pluriverse to form a ring. Finally the schemas are not just a self-produced ring, but they are also socially projected as we all see the same schemas even though we may disagree about what schema to apply in each case to each ontic phenomena and also disagree about the kind to apply to the specific schematic envelope. Schemas are a socially created, or invented, and constructed first classification of phenomena that is socially produced. If there is reflexive mirroring then the schemas are the phalanges of these mirrors. And these phalanges are determined in large part by the generalization of the integer line called the Pascal Triangle and its extensions and simplicies. So there is an intimate relation between General Schemas Theory and the Generalized Integer Line that we call the Pascal Triangle. Part of that generalization is the extension into the negative and imaginary dimensions and their concrete realization though the Basic Mathematical Categories which are comprised of Set, Mass, Reserve and Field and their interactions with respect to Etter's Link Theory.